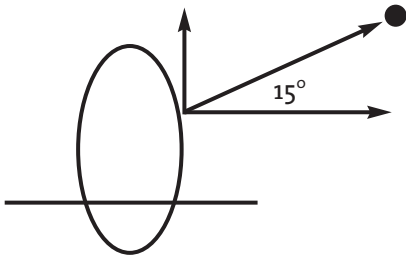## Answer Key

Firing the Cannons on *Constitution*, using parabolic equations & vectors to describe the path of projectile motion

A cannonball is fired from the spar deck of USS Constitution at an initial velocity of 1,500 m/s. The cannon was inclined 15° and positioned 32 ft from the water.

1. Draw a quick sketch of the situation.

2. Calculate the vertical and horizontal components of the velocity vector.

v_y = 1500 sin 15 ≈ 388 *m/s*

v_x = 1500 cos 15 ≈ 1449 *m/s*

3. Substitute the values into the equations $y(t) = \frac{-1}{2}gt^2 + v_y t + h_o$, the equation for vertical distance and $x(t) = v_x t$, the equation for horizontal distance.

$y(t) = \frac{-1}{2}(32)t^2 + 388t + 32 = -16t^2 + 388t + 32$

$x(t) = 1449t$

4. How high is a cannonball after 3 seconds? How far has it traveled?

height: $y(3) = -16(3)t^2 + 388(3) + 32 = 1052$ *ft*

distance: $x(3) = 1449(3) = 4347$ *ft*

5. What is the maximum height of the cannonball? How long does it take to reach this height? How far has it traveled in this time?

time to reach maximum: $t = \dfrac{-b}{2a} = \dfrac{-388}{2(-16)} = \dfrac{-388}{-32} = 12.25s$

maximum height: $h(12.125) = -16(12.125)^2 + 388(12.125) + 24 = -2352.25 + 4704.5 + 24 = 2376.25$

horizontal distance from boat: $x(12.125) = 1449(12.125) = 17750.25\,ft$

6. Assume that the cannonball misses its target. When will the cannonball hit the water? How far has it traveled?

calculate the zeros:

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-388 \pm \sqrt{388^2 - 4(-16)(32)}}{2(-16)} \approx \dfrac{-388 \pm 391}{-32} \rightarrow t = 24.3$

horizontal distance from boat: $x(24.3) = 1449(24.3) = 35210.7\,ft$

7. Could the cannons hit a target 2 miles away? (1 mile = 5,280 feet) How long would it take for the cannonball to reach its target?

Based on the math, yes because the cannonball could roughly reach 6 miles. This could lead into a good discussion about reality of this situation.

- What forces may slow the cannonball down?
- How accurate is our information and the accuracy of the cannon?

It would take roughly 7.3 seconds to reach the target.